Multi-agent Pathfinding on Large Maps Using Graph Pruning: This Way or That Way?

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Abstract: Multi-agent pathfinding is the task of navigating a set of agents in a shared environment from their start locations to their desired goal locations without collisions. Solving this problem optimally is a hard task and various algorithms have been devised. The algorithms can generally be split into two categories, search- and reduction-based ones. It is known that reduction-based algorithms struggle with large instances in terms of the size of the environment. A recent study tried to mitigate this drawback by pruning some vertices of the environment map. The pruning is done based on the vicinity to a shortest path of an agent. In this paper, we study the effect of choosing such shortest paths. We provide several approaches to choosing the paths and we perform an experimental study to see the effect on the runtime.

1 Introduction

We study the problem of Multi-agent pathfinding (MAPF). The task is to navigate a set of agents in a shared environment (map) from starting locations to the desired goal locations such that there are no collisions (Silver, 2005). This problem has numerous practical applications in robotics, logistics, digital entertainment, automatic warehousing and more, and it has attracted significant focus from various research communities in recent years (Li et al., 2020; Surynek, 2019; Nguyen et al., 2017; Gebser et al., 2018b).

Optimal MAPF solvers can generally be split into two categories, search- and reduction-based ones. The former search over possible locations or conflicts among agents, while the latter reduce the problem to other formalisms, such as Answer Set Programming (ASP) (Gebser et al., 2018a). While it is not always the case, it is generally established that each of the approaches dominates on different types of instances (Gómez et al., 2021; Svancara and Barták, 2019). The search-based solvers are easily able to find solutions on large sparsely populated maps while having trouble dealing with small densely populated maps. On the other hand, the reduction-based solvers are able to deal with the small densely populated maps but are unable to find a solution for large maps even with a small number of agents.

Since reduction-based solvers have trouble solving large instances, a recent study (Husár et al., 2022) proposed techniques to remove vertices from the map that are most likely not needed to solve the instance. The pruning is done based on a random shortest path for each agent. Only vertices around the selected path are considered and other vertices are removed, creating a much simpler problem. In this paper, we extend the original study by examining the behavior of the technique by using more than just one shortest path for each agent. First, we describe the four pruning strategies from the original study, and then we introduce four new ones to select paths for each agent.

2 Definitions

A *MAPF instance* $\mathcal{M}$ is a pair (G,A), where G is a graph $G=(V,E)$ and A is a set of agents. An agent $a_i \in A$ is a pair $a_i=(s_i,g_i)$, where $s_i \in V$ is the start location and $g_i \in V$ is the goal location of agent a_i.

Our task is to find a *valid plan* π_i for each agent $a_i \in A$ being a valid path from s_i to g_i. We use $\pi_i(t)=v$ to denote that agent a_i is located at vertex v

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at timestep t. Time is discrete and at each timestep, an agent can either wait at its current location or move to a neighboring location. Furthermore, we require that each pair of plans π_i and π_j, $i \neq j$ is collision-free. Based on MAPF terminology (Stern et al., 2019), there are five types of collisions. Here, we forbid *edge*, *vertex*, and *swap* conflicts while allowing *follow* and *cycle* conflicts since the two latter prevent agents from occupying the same location. Note, however, that all of our methods work in any setting.

We are interested in *makespan* optimal solutions. Makespan (or horizon) refers to the length of a plan. A plan ends once all of the agents are at the goal location at the same time. This means that the length of the plan $|\pi_i|$ is the same for all of the agents. Another common cost function is *sum of costs* (Sharon et al., 2011). Note that finding an optimal solution for either of the cost functions is an NP-hard problem (Ratner and Warmuth, 1990; Yu and LaValle, 2013).

3 ASP Encoding

We use an ASP encoding[1] for MAPF due to (Gebser et al., 2018a). The encoding assumes a grid graph G and plans agents in parallel within a makespan while avoiding conflicts. Specifically, the plan's timesteps are bound by the makespan H in Line 1. Line 3 gives the four cardinal directions, used in Line 4 to represent all transitions on the grid with its x,y-coordinates stated by predicate `position/1`. Possible movement actions, at most one per agent and timestep, are generated by Line 8. Related preconditions and positional changes are described in Lines 10-12: The positions of all agents are described by `position(R,C,T)` stating that agent `R` is at x,y-coordinates `C` at time `T`. For an agent `R` sitting idle at time `T`, the frame axiom in Lines 14-15 propagates its unchanged position. Swapping conflicts are prevented by Lines 17-19, and both edge and vertex conflicts by Line 21.

```
1  time(1..horizon).

3  direction((X,Y)) :- X=-1..1, Y=-1..1
     , |X+Y|=1.
4  nextto((X,Y),(DX,DY),(X',Y')) :-
5      direction((DX,DY)), position((
         X,Y)), position((X',Y')),
6      (X,Y)=(X'-DX,Y'-DY), (X',Y')=(X+
         DX,Y+DY).

8  { move(R,D,T) : direction(D) } 1 :-
     isRobot(R), time(T).

10 position(R,C,T) :-
11     move(R,D,T), position(R,C',T-1),
           nextto(C',D,C).
12  :- move(R,D,T), position(R,C ,T-1),
       not nextto(C ,D,_).

14 position(R,C,T) :-
15     position(R,C,T-1), not move(
          R,_,T), isRobot(R), time(T).

17 moveto(C',C,T) :-
18     nextto(C',D,C), position(R,C',T
          -1), move(R,D,T).
19  :- moveto(C',C,T), moveto(C,C',T),
       C < C'.

21  :- { position(R,C,T) : isRobot(R) }
          > 1, position(C), time(T).
```

Listing 1: Action theory for agent movements.

Further, we augment the action theory encoding with the goal condition in Listing 2 to enforce that every agent `R` has reached its goal coordinates `C`, stated by `goal(R,C)`, at the time H.

```
 :- not position(R,C,horizon), goal(
     R,C).
```

Listing 2: Goal condition for agents and assigned nodes.

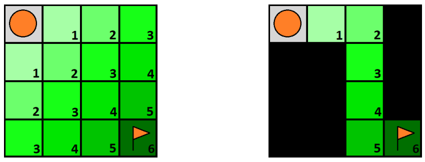
There are two common techniques to speed up computation. First, using a lower bound for the makespan. A simple lower bound is to compute for each agent a_i the shortest path from its start location s_i to its goal location g_i. The lower bound for H is then the longest of these shortest paths. Another enhancement is to preprocess the variables representing agent locations. These variables correspond to an agent being present at some location at a time. However, for some locations, the specific agent cannot be present at the specific times. Specifically, for agent a_i, if vertex v is distance d away from start location s_i, we know that the agent a_i cannot be at vertex v at times $0,\ldots,(d-1)$. Similarly, if vertex v is distance d away from goal location g_i, agent a_i cannot be present at vertex v at times $H-d+1,\ldots,H$.

4 The Sub-Graph Method

Both of the above improvements maintain completeness and optimality. However, there are situations, where too many possibilities for an agent's location remain, which may overwhelm the underlying solver. The motivation of (Husár et al., 2022) can be seen in Figure 1a. The agent is placed on a 4-connected grid going from one corner to the diagonally opposite corner. With just one agent and no obstacles, there are $\binom{2(N-1)}{N-1}$ possible shortest paths on an $N \times N$ grid. As shown in the figure, preprocessing finds out at what timesteps the agent can be located at which vertices.

[1] https://github.com/potassco/asprilo-encodings/blob/master/m/action-M.lp

However, the number of choices is still too large for the solver. The idea is to pick just one of the shortest paths and to treat the other vertices as impassable obstacles. So, for these vertices, no variables enter the solver. This pruning is shown in Figure 1b.



(a) (b)

Figure 1: An agent moving on a grid map from a corner to the opposite one. The numbers represent at what timesteps the agent can reach the given vertex.

Of course, this pruning does not maintain completeness in general. A simple counterexample is given in Figure 2. The two agents want to swap their location (ie. their goal location is identical with the starting location of the other agent). To do this, the only solution is for both of them to travel to the right and use the top vertex to switch their position. To mitigate these instances, several strategies are proposed to change which vertices are pruned.

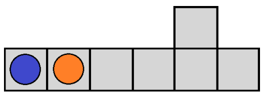


Figure 2: An instance with two agents that want to swap their positions.

Solving Strategies

Let SP_i be the set of vertices on a chosen shortest path for agent $a_i \in A$ (ie. a single shortest path from s_i to g_i). The length of the path is $|SP_i|$. The union of vertices on the shortest paths of all agents is $SP_A = \bigcup_{a_i \in A} SP_i$. Note that we consider a single shortest path for each agent. If multiple shortest paths exist for an agent, one is chosen at random. Given this, the lower bound on the makespan of an instance (G,A) is $LB_{mks}(G,A) = \max_{a_i \in A} |SP_i|$. For short, we refer to such lower bound just by LB.

A *k-restricted graph* $Gres_k^{SP_A}$ is a subgraph of G containing only vertices SP_A and vertices at most distance k away from some vertex in SP_A. Since we always fix SP_A, we write for simplicity only $Gres_k$. Note that $Gres_k \subseteq Gres_{k'}$ for $k \leq k'$.

We define a *makespan-restricted* MAPF instance as $\mathcal{M} = (G,A,H)$. A makespan optimal solution is found by iteratively increasing the makespan. The *(k,m)-relaxation* of $\mathcal{M}$ is the makespan-restricted MAPF instance

$$\mathcal{M}_{k,m} = (Gres_k, A, LB + m)$$

This relaxation considers only $Gres_k$ instead of the whole graph G. We find a solution with extra makespan m – extra over the lower bound on makespan. Also note that $Gres_k$ is constructed such that $LB_{mks}(G,A) = LB_{mks}(Gres_k,A)$ for any k, therefore, we do not need to change the notation of LB.

We build a partial order $\prec_{relax}$ over the (k,m)-relaxations $\mathcal{M}_{k,m}$ such that

$$\mathcal{M}_{k,m} \prec_{relax} \mathcal{M}_{k',m'}$$

if $k \leq k'$, $m \leq m'$ and $k+m < k'+m'$

There is an upper bound on k such that for some k_{max} we have $Gres_{k_{max}} = G$. There is also an upper bound on the makespan for a given MAPF instance of $O(V^3)$ (Kornhauser et al., 1984). For example, assume that $k_{max} = 3$ and $m_{max} = 2$. Then, Figure 3 depicts the space of possible relaxations induced by $\prec_{relax}$. Note that the partial ordering forms a lattice.

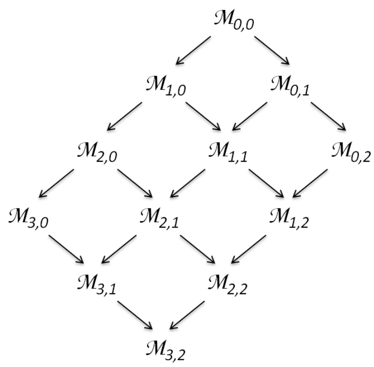


Figure 3: Instance relaxations for $k_{max} = 3, m_{max} = 2$.

The generic algorithm to solve MAPF using the relaxed instances is as follows. First, we build an initial (k,m)-relaxation and we iteratively change k and m until the instance is solvable. This corresponds to a traversal of the lattice formed by the partial ordering $\prec_{relax}$. Note that the shortest path for each agent is fixed for all of the iterations.

Next, we identify four reasonable traversals.

Baseline Strategy. The classical approach to solving MAPF makespan optimally can be expressed in the relaxed instances as follows. We start with an initial candidate of k_{max} (ie. the whole graph G) and $m = 0$. If the relaxed instance is unsolvable, only the additional makespan m is increased to $m+1$. We shall refer to this strategy as *baseline* or **B** for short.

Proposition 1. *If a MAPF instance $\mathcal{M}$ has a solution,* baseline *strategy finds an optimal solution.*

Makespan-add Strategy. The first smarter solution is to keep only the vertices on the shortest paths and the immediately adjacent ones. The initial candidate is $k = 1$ and $m = 0$. Otherwise, the strategy is the same as the baseline strategy: if the relaxed instance is unsolvable, we increase m to $m+1$ while k is never changed. We refer to this strategy as *makespan-add* or **M** for short.

Proposition 2. *(Husár et al., 2022) The* makespan-add *strategy is both suboptimal and incomplete.*

On the other hand, in most cases, this simple strategy can find a solution, and due to the great reduction of vertices of the graph, the solution may be found quickly. We choose to start with $k = 1$ rather than $k = 0$ to increase the probability for a solution to exist while keeping the number of vertices to a minimum.

Prune-and-cut Strategy. The previous strategies either use unnecessary large restricted graph or do not guarantee to find a solution. Strategy *prune-and-cut* (**P** for short) guarantees both completeness and optimality. We start with initial candidate $k = 0$ and $m = 0$. In case the relaxed instance is unsolvable, we cannot be sure if the reason is the restriction on k or on m. However, since we do not want to overestimate m, we first need to increase k potentially up to k_{max}. Once a restricted instance $\mathcal{M}_{k_{max},m}$ is unsolvable, we are sure that m needs to be increased. We can optimistically assume that the whole $Gres_{k_{max}}$ is not needed and we restrict the graph back to $k = 0$ producing $\mathcal{M}_{0,m+1}$.

Proposition 3. *(Husár et al., 2022) If a MAPF instance $\mathcal{M}$ has a solution,* prune-and-cut *strategy finds an optimal solution.*

Combined Strategy. The drawback of the *prune-and-cut* strategy is that in the case the makespan needs to be increased, we first increase k up to k_{max} before increasing m. To mitigate this problem, we present the *combined* strategy (**C** for short). The initial candidate is again $k = 0$ and $m = 0$. If the relaxed instance is unsolvable, we increase both $k = k+1$ and $m = m+1$ at the same time. This way, we save solver calls because we do not need to explore all of the possible reductions in the k direction. On the other hand, this strategy is no longer optimal. Figure 4 with blue agent choosing the blue path is a counterexample.

Proposition 4. *(Husár et al., 2022) If a MAPF instance $\mathcal{M}$ has a solution,* combined *strategy is guaranteed to find a solution (completeness) but not necessarily an optimal one.*

5 Choosing the Shortest Paths

The described strategies (except for *baseline*) may suffer from a poor choice of the initial shortest path for each agent. See the example in Figure 4. The blue agent has two possible shortest paths. If the algorithm by random chooses the blue path, none of the sophisticated strategies can solve the relaxed instance in the first solver call. *Makespan-add* would find a suboptimal solution with makespan $LB+2$, *prune-and-cut* would require to increase k two times to be able to use the black path, and *combined* strategy would also find a suboptimal solution with makespan $LB+2$.

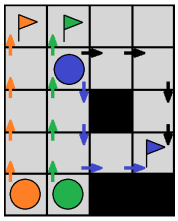


Figure 4: An example instance where the blue agent has two choices of the shortest path. If the blue path is chosen, the proposed strategies perform worse.

This issue can be mitigated by including all of the vertices on all of the possible shortest paths into the $Gres_k$, however, this goes against the logic of the motivational example in Figure 1. Hence, we try to identify approaches to choose more than just one of the shortest paths to improve the strategies.

Since the choice of the shortest paths acts as a preprocessing stage, we aim for fast heuristic techniques. For this reason, each agent is treated individually, without considering the interference with shortest paths of the other agents. We propose the following four sensible approaches to pick which vertices should be included in the initial restricted graph $Gres_0$. All of the described strategies, then, work the same as was described in the previous section.

Single Path. First, we use the same approach as in the original study (Husár et al., 2022). For each agent, we choose a single random shortest path. The restricted graph $Gres_0$ is induced by $SP_A = \bigcup_{a_i \in A} SP_i$. Recall that SP_i are the vertices on the shortest path for agent a_i. We refer to this approach as *single-path* or **SP** for short.

All Paths. The second approach is on the other end of the spectrum. Instead of just one shortest path, we consider all vertices on all shortest paths of a given agent. Formally, we write $SP_i^{All} = \{v \in V \mid dist(s_i, v) + dist(v, g_i) = |SP_i|\}$ meaning all vertices whose distance from start location plus the distance to goal location equals the distance of a shortest path. The restricted graph $Gres_0$ is induced by $SP_A^{All} = \bigcup_{a_i \in A} SP_i^{All}$.

Note that while there may be many different shortest paths as discussed in Figure 1, the number of vertices on those paths is much smaller. For the creation of the restricted graph, we are interested only in the

(a) The greedy approach starting at s_i chooses an undesirable green path due to the fact that it tries to make the path most divers (to the previously planned blue and red paths) from the start without the knowledge of the rest of the map.

(b) By choosing a different starting location, the greedy algorithm finds a better green path than in Figure 5a. In this example there are multiple possible starting locations, each equally good. If this happens, one is chosen at random.

Figure 5: An example showing the drawback of finding the shortest path greedily from the the starting location s_i. This issue can be fixed by choosing different starting vertex. The green path is chosen after red and blue paths.

vertices, the specific path is decided by the underlying solver. Finding all of the vertices on all of the shortest paths can be done by performing a breath-first search from the start and goal of the agent and checking for the condition in the definition of SP_i^{All}. We refer to this approach as *all-paths* or **AP** for short.

Random Paths. Instead of considering one or all possible paths, we aim to pick vertices that are part of just some subset of all paths. First, we need to set a number of paths to consider. Note that based on the given map and the start and goal locations of each agent, there is a wide variety of the number of shortest paths. Instead of selecting a magic constant, we propose to find $\frac{|SP_i^{All}|}{|SP_i|}$ shortest paths for agent a_i and consider the union of vertices on those. If there is a unique shortest path, by using the formula we correctly consider just the one shortest path, while on an empty $N \times N$ grid (such as in Figure 1), we are considering $\frac{N}{2}$ paths.

The next proposed approach picks the specified number of shortest paths randomly. We do this by a random walk starting at s_i moving only over vertices from SP_i^{All} in the correct direction. We know the correct direction based on the distance from s_i and g_i computed by BFS (we need to perform the two BFS in order to determine SP_i^{All}). The random walk is biased to prefer vertices that are not yet used for a given agent. By doing this for all agents we get $SP_A^{Rand} = \bigcup_{a_i \in A} SP_i^{Rand}$. We refer to this approach as *random-paths* or **RP** for short.

Distant Paths. The drawback of *random-paths* it that there is no guarantee on the properties of the chosen shortest paths. The idea of using more than one shortest path is to allow the underlying solver to navigate the agent through a different region of the map to avoid possible conflicts. However, by choosing random paths, we may produce paths that share many vertices or are in close proximity to each other, both of which are undesirable.

We want to find *diverse* and *distant* paths. There is a polynomial-time algorithm to find diverse shortest paths (Hanaka et al., 2021). In this case, diverse means paths that share the least amount of edges (or vertices). By using this algorithm, it may be the case that we find paths as shown in Figure 6. On the other hand, there is also a research dealing with finding the most diverse near-shortest paths (Häcker et al., 2021), in which case the paths are supposed to be the greatest distance from each other. Note that both studies use the term distant with different meaning. In our paper, we are using *diverse* for different paths and *distant* for path with distance between them. The downside of the the second referred study is that the paths found are not optimal and also the problem itself is NP-Hard, which is not a desirable trait for a preprocessing function.

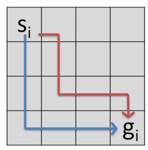


Figure 6: Possible shortest paths from s_i to g_i found by the diverse shortest paths algorithm (Hanaka et al., 2021).

Our proposed approach is heuristic. Again, we build the paths over the vertices from SP_i^{All}, gradually creating SP_i^{Dist}. At each step, we try to add a new vertex to the currently build path and if there are multiple choices, we pick one that maximizes the minimal distance to all of the vertices currently in SP_i^{Dist} (see Figure 7 for an example). Since this is just a heuristic, there are examples that make us choose an undesirable path because the approach greedily chose the next vertex on the path without knowledge of the rest of the map. Such example can be seen in Figure 5a. To mitigate this, we start to build the path from a different vertex from the set SP_i^{All} rather than from s_i. The first path is build the same as SP_i, for the latter paths, we choose a vertex v such that it maximizes the minimal distance to all of the vertices currently in SP_i^{Dist}. This way we need to build the path both from

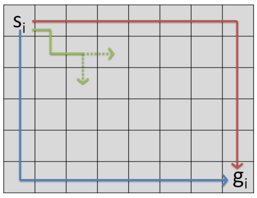


Figure 7: The gradual building of SP_i^{Dist} from s_i to g_i by a greedy approach. The currently build green path has a choice. Moving downward will be chosen since it maximizes the distance to the already chosen paths.

v to s_i and from v to g_i. Using this approach we find a much more desirable path for the example in Figure 5a with the result shown in Figure 5b. We refer to this approach as *distant-paths* or **DP** for short.

6 Experimental Evaluation

To test and compare the proposed strategies in combination with approaches to creating the restricted graph, we set up experiments. The full implementation and results are available at `https://github.com/potassco/mapf-subgraph-system`. For the ASP-based solver, we used the grounding-and-solving system *clingo* (Gebser et al., 2019; Kaminski et al., 2020) version 5.5.2. We ran the experiments on an Intel Xeon E5-2650v4 under Debian GNU/Linux 9, with each instance limited to 300s processing time and 28 GB of memory.

The instances used in our experiments are based on commonly used benchmark instances available online (Stern et al., 2019). We chose different sizes of maps – *small* (32 by 32), *medium* (64 by 64), and *large* (128 by 128) and different structures of the impassable obstacles in the map with the following types – *empty*, *maze*, *random*, and *room*.

For the placement of the agents (called *scenarios*), we used the available scenarios. Furthermore, we created new scenarios for each map such that the distance from start to goal of each agent is similar and the paths of the agents need to cross more often. We did this because the makespan optimal solution for the random scenarios rarely differs from the lower bound. The behavior of the strategies may be gravely affected by many conflicts and the need to increase the makespan.

The intended way to use the benchmark set is to create an instance of MAPF from a map and a number of agents from a scenario. If the instance is solved in the given time limit, an additional agent from the same scenario is added and thus a new MAPF instance is produced. Once the instance cannot be solved in the time limit, it is reasoned that increasing further the number of agents cannot make the instance solvable. We are aware that using a reduction-based solver, this may not always hold. Also, some of the strategies may benefit from additional agents which change the restricted graph. However, these cases are extremely rare and therefore, we decided to use the benchmark as intended.

Table 1 shows the results for all of the strategies and approaches to creating the restricted graph. Note that the *baseline* strategy **B** considers the whole map, therefore we do not use any of the four approaches. The strategies **B** and **P** are optimal, therefore, we consider them separately opposed to the suboptimal strategies **C** and **M**. The best result for both optimal and suboptimal strategies on each line is highlighted. We present the results divided by the type of the map regardless of the size. This representation shows nicely the difference between the approaches to creating the restricted graph. For more detailed results, we include much more detailed tables in the supplementary materials.

First, we examine the average number of vertices used by each approach. In the table, the number indicates the ratio of used vertices to the total number of vertices. Since **B** always uses the whole map, the ratio is 1. We can see that **SP** uses the least number of vertices in all cases, on the other hand, **AP** uses the most and **DP** and **RP** use about the same. This result is not surprising since it is based on the number of paths used by each approach. However, we can also see that the difference is much bigger on opened maps (such as *empty*) and much smaller on very restrictive maps (such as *maze*), meaning that in the latter case there are not many different shortest paths for the agents to choose from. There is also a clear order in terms of the strategies with **P** using the least, **C** using more, and **M** using even more vertices on average.

Examining the number of solved instances (ratio of solved to all instances – 2544 for *empty*, 1956 for *maze*, 2418 for *random*, 2357 for *room*), we see that the most successful combination is **P** + **SP** for the optimal setting and **C** + **SP** for the suboptimal. Again, the difference across the approaches to choosing the shortest paths is least prominent on *maze* maps, however, on the other types, the order is clear. The **SP** is the most successful, **DP** and **RP** performing about the same, while **AP** performs the worst. The *baseline* **B** performs worse than any other used combination. Similar results can be seen when exploring the IPC score [2] (Computed as 0 if the solver did not finish in time, otherwise as $\frac{\textit{min. time}}{\textit{solver time}}$, where *min. time* is the time it took the fastest solver and *solver time* is the time it took the solver in question. The score ranges from 0 to 1, where the bigger the number the bet-

[2] introduced at International Planning Competition.

	type	B	P				C				M			
			SP	AP	RP	DP	SP	AP	RP	DP	SP	AP	RP	DP
Used vertices	*empty*	1	**0.14**	0.23	0.21	0.23	**0.15**	0.24	0.22	0.23	0.19	0.24	0.23	0.24
	maze	1	**0.18**	0.2	**0.18**	0.19	**0.20**	0.22	0.21	0.21	0.22	0.22	0.22	0.22
	random	1	**0.19**	0.27	0.24	0.25	**0.22**	0.3	0.28	0.29	0.25	0.31	0.3	0.31
	room	1	**0.21**	0.24	0.22	0.22	**0.23**	0.27	0.25	0.25	0.24	0.29	0.27	0.28
Solved instances	*empty*	0.78	**0.99**	0.81	0.84	0.82	**1.00**	0.81	0.84	0.82	0.87	0.81	0.82	0.8
	maze	0.85	0.87	**0.88**	0.87	0.87	**0.98**	0.97	0.97	**0.98**	0.94	0.94	0.94	0.94
	random	0.79	**0.91**	0.82	0.84	0.84	**1.00**	0.89	0.92	0.91	0.93	0.87	0.89	0.88
	room	0.8	**0.83**	0.81	0.82	0.82	**0.97**	0.92	0.95	0.94	0.89	0.89	0.89	0.89
$\sum$ IPC	*empty*	874.6	**1930.5**	1312.8	1081.7	1086.3	**2395.2**	1276.5	1406.7	1324.4	1517.9	1275.8	1312.5	1279.2
	maze	890.1	**1153.2**	1123.6	1128.3	1143.3	**1775.5**	1633.8	1752.8	1754	1475.7	1452.6	1470.5	1466.1
	random	886	**1668.0**	1110.2	1049.9	1102.3	**2275.5**	1422.4	1594.6	1551.5	1560.7	1300.3	1374.2	1324.4
	room	877.3	**1386.4**	1007.5	1191.6	1179.5	**2078.3**	1594.5	1850.6	1828.7	1474	1382.6	1461.9	1442
	total	3527.9	**6138.1**	4554.1	4451.6	4511.4	**8524.5**	5927.2	6604.6	6458.6	6028.4	5411.3	5619.2	5511.6
Solved optimally	*empty*	-	-	-	-	-	0.93	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
	maze	-	-	-	-	-	0.91	**0.94**	0.92	0.92	0.92	0.93	0.93	0.93
	random	-	-	-	-	-	0.89	**0.97**	0.95	0.96	0.86	0.91	0.91	0.91
	room	-	-	-	-	-	0.86	**0.93**	0.91	0.92	0.76	0.86	0.85	0.85
Conflicts	*empty*	133	**97**	148	131	136	**111**	147	141	142	139	148	146	148
	maze	2457	**1239**	1804	1274	1265	3101	**2969**	3120	3143	3361	3482	3443	3354
	random	206	193	195	174	**173**	**218**	235	234	227	448	458	460	459
	room	1007	402	313	270	**289**	1642	1414	1513	1423	1525	**1400**	1527	1545
Constraints [millions]	*empty*	**4.7**	7.3	5.1	5.9	5.2	7.4	5.1	6	5.2	6	**5.0**	5.2	**5.0**
	maze	**6.0**	6.1	6.4	6	6.2	**6.4**	6.8	6.5	6.6	6.8	6.8	6.8	6.8
	random	**5.4**	5.9	6	6	6	6	6.1	6.1	6.1	6.1	**5.8**	6	**5.8**
	room	**4.8**	5.4	5.6	5.4	5.4	**5.7**	5.9	5.8	**5.7**	**5.7**	5.9	5.9	5.9

Table 1: Ratio of used vertices, ratio of solved instances, sum of IPC score, ratio of instances solved optimally, average number of conflicts, and average number of constraints. The results are split by the map type. Strategies are *baseline* (**B**), *prune-and-cut* (**P**), *makespan-add* (**M**), and *combined* (**C**). Approaches to choosing shortest paths are *single-path* (**SP**), *all-paths* (**AP**), *random-paths* (**RP**), and *distant-paths* (**DP**).

ter. The scores of all instances are summed.) For the **P** strategy the **AP** approach performs better than **DP** and **RP**, meaning that while it did not solve more instances, the instances it managed to solve were solved faster. For the other strategies, the order remains the same as with the number of solved instances. It is unsurprising that the suboptimal strategies achieved a better score that the optimal **P**.

We argue that these results stem from the number of used vertices. By exploring the ASP solver, we see that for all strategies and all additional shortest path approaches, the number of conflicts stays mostly within the same order of magnitude as for **SP**. Hence, ASP search difficulty remains unchanged. However, compared to **SP**, the other approaches add more vertices to the restricted graph to consider and, in consequence, this increases the grounding time of *clingo* which, in turn, leads to more timeouts.

The new shortest path approaches reduce the size of the internal problem specification in terms of the number of constraints. We conjecture that since the new approaches generally select multiple (and more likely exclusively usable by one agent) vertices for the restricted graph, the amount of constraints encoding possible agent collisions is reduced. However, as mentioned above, this has no significant impact on the search complexity.

We also explore the quality of the solutions produced by the suboptimal strategies. The ratio of instances solved optimally is again shown in Table 1. Strategy **C** is more often optimal compared to **M**. This time, we can see the benefit of adding extra vertices to the restricted graph. The most often optimal approach is **AP** closely followed by **RP** and **DP**, while **SP** achieved the worst results. The difference is again less prominent on *maze* maps.

7 Conclusion

We extended the study on pruning maps to increase the efficiency of reduction-based MAPF solvers. In the original paper, only one random path was chosen for each agent to build a restricted graph. Conversely, in this paper, we proposed several approaches to choosing multiple different paths for each agent, providing the underlying solver with more choices. In theory, this should make it possible for the agents to avoid collisions more easily. In our experiments, we found that this rarely happens and that it is more beneficial to provide the solver with just one random path making the relaxed instances simpler for the cost of possibly having to solve more relaxations. Thus, we showed that the original approach is justified, a result that is lacking in the original study. On the other hand, we also showed that providing the agents with more possible paths leads more often to an optimal solution when using one of the suboptimal strategies.

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