

Improving the Sum-of-Cost Methods for Reduction-based Multi-agent Pathfinding Solvers

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Abstract: Multi-agent pathfinding is the task of guiding a group of agents through a shared environment while preventing collisions. This problem is highly relevant in various real-life scenarios, such as warehousing, robotics, navigation, and computer games. Depending on the context in which the problem is applied, we may have specific criteria for the quality of a solution, expressed as a cost function. The most common cost functions are the makespan and sum-of-cost. Minimizing either of them is computationally challenging, leading to the development of numerous approaches for solving multi-agent pathfinding. In this paper, we explore reduction-based solving under the sum-of-cost objective. We introduce a reduction to answer set programming (ASP) using two existing approaches for sum-of-cost minimization, originally introduced for a reduction to Boolean satisfiability (SAT). We propose several enhancements and use the Clingo ASP system to implement them. Experiments show that these enhancements significantly improve performance. Particularly, the performance on larger maps increases in comparison to the original variants.

1 Introduction

Multi-agent pathfinding (MAPF) is the task of guiding a group of agents through a shared environment while preventing collisions. This problem is highly relevant in various real-life scenarios, such as warehousing (Ma et al., 2017), robotics (Bennewitz et al., 2002), navigation (Dresner and Stone, 2008), and computer games (Wang and Botea, 2008).

Depending on the context, we may have specific criteria for the quality of the solution, expressed as a cost function. The most commonly used cost functions are makespan (i.e., minimizing the time for all agents to reach their destinations) and sum-of-cost (i.e., minimizing the total number of actions performed by all agents) (Stern et al., 2019). Each of these cost functions serves a practical purpose: makespan optimization focuses on minimizing the overall task completion time, even if it means some agents perform more actions. On the other hand, sum-of-cost optimization aims to reduce the total number of actions, which can be associated with minimizing energy consumption.

Minimizing either of these cost functions is computationally challenging (Yu and LaValle, 2013), leading to the development of numerous approaches for optimally solving MAPF problems (Boyarski et al., 2015;

Lam et al., 2019; Surynek et al., 2016; Sharon et al., 2011). In this paper, we explore reduction-based solving under the sum-of-cost objective. We review the two existing optimization approaches – *iterative* (Surynek et al., 2016) and *jump* (Barták and Švancara, 2019), both originally developed for reductions to Boolean satisfiability (SAT). Then, we introduce a reduction to answer set programming (ASP) (Gebser et al., 2012; Lifschitz, 2019) and adapt the two optimization approaches. Furthermore, we propose several enhancements to the jump approach and leverage the multi-shot solving capabilities and inbuilt optimization strategies of the Clingo (Kaminski et al., 2023) ASP system to implement them. The first enhancement is the use of techniques from the iterative approach to quickly find an initial solution. Next, we improve on the first enhancement by increasing the iteration step size. Finally, we show unsatisfiable-core based optimization dramatically improves performance. We present a series of experiments demonstrating that these enhancements indeed lead to performance gains. Particularly, the performance on larger maps is increased in comparison to the original variants.

2 BACKGROUND

The *multi-agent pathfinding* problem (MAPF) (Stern et al., 2019) is a pair (G, A) , where G is an undirected graph $G = (V, E)$ and A is a list of agents $A = (a_1, \dots, a_n)$. Each agent $a_i \in A$ is associated with a start vertex $s_i \in V$ and a goal vertex $g_i \in V$.

Time is considered discrete; between two consecutive timesteps, an agent can either move to an adjacent vertex (move action) or stay at its current vertex (wait action). The movement of an agent is captured by its path. A *path* π_i of agent a_i is a list of vertices that starts at s_i and ends at g_i . Let $\pi_i(t)$ be the vertex (i.e., location) of a_i at timestep t according to π_i . Therefore, $\pi_i(0) = s_i$, $\pi_i(|\pi_i|) = g_i$, and for all timesteps $t < |\pi_i|$, $(\pi_i(t), \pi_i(t+1)) \in E$ or $\pi_i(t) = \pi_i(t+1)$, that is, at each timestep agent a_i either moves along an edge or waits at a vertex, respectively.

As there are several agents, we are interested in the interaction of pairs of paths of distinct agents. There is a *conflict* between paths π_i and π_j at timestep t if $\pi_i(t) = \pi_j(t)$ (*vertex conflict*) or $\pi_i(t) = \pi_j(t+1)$ and $\pi_j(t) = \pi_i(t+1)$ (*swapping conflict*). A *plan* Π is a list of n paths $\Pi = (\pi_1, \dots, \pi_n)$, one for each agent. A *solution* is a conflict-free plan, i.e., a plan Π where no two paths of distinct agents have conflicts.

A solution is *optimal* if it has the lowest cost among all possible solutions. The cost $C(\pi_i)$ of path π_i equals the number of actions performed in π_i until the last arrival at g_i , not counting any subsequent wait actions. Formally, $C(\pi_i) = \max(\{0 < t \leq |\pi_i| \mid \pi_i(t) = g_i, \pi_i(t-1) \neq g_i\} \cup \{0\})$. Note that waiting at the goal counts towards the cost if the agent leaves the goal at any time in the future.

There are two commonly used cost functions to evaluate the quality of a plan Π :

1. *sum-of-costs* (SOC), which is the sum of costs of all paths $C_{SOC}(\Pi) = \sum_i C(\pi_i)$
2. *makespan* (MKS), which is the maximum cost among all paths $C_{MKS}(\Pi) = \max_i C(\pi_i)$.

Although the decision problem of whether a MAPF problem has a solution is polynomial (Kornhauser et al., 1984), bounding the movement of the agents turns it into an NP-complete problem (Yu and LaValle, 2013; Surynek, 2010). This makes finding an optimal solution much harder than finding any solution. Figure 1 presents a MAPF problem instance with two agents a_1 and a_2 , with start vertices s_1 and s_2 , and goal vertices g_1 and g_2 , respectively. Here, the optimal solution minimizing the sum-of-costs is $\pi_1 = (s_1, E, F, G, H, I, g_1)$ and $\pi_2 = (s_2, B, A, g_2)$, which yields $C_{SOC}(\Pi) = 9$ and $C_{MKS}(\Pi) = 6$. The optimal solution minimizing the makespan is $\pi_1 = (s_1, A, B, C, D, g_1)$ and $\pi_2 = (s_2, s_2, s_2, B, A, g_2)$, which

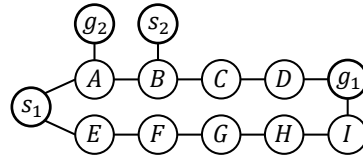


Figure 1: Example of a MAPF problem instance with different SOC and makespan optimal solutions.

yields $C_{SOC}(\Pi) = 10$ and $C_{MKS}(\Pi) = 5$. This example illustrates that both cost functions are indeed different and optimizing one may increase the other.

3 FINDING OPTIMAL SOLUTIONS

The basic idea in a reduction-based approach for solving MAPF is to model agents' positions in time. The correct number of timesteps in which the agent may perform its moves has to be found.

As explained in Section 2, there are two common cost functions to evaluate the quality of a plan. Finding *makespan* optimal solutions is straightforward, as it follows a basic iterative deepening approach (Surynek et al., 2016); the makespan is incremented until the bounded problem becomes satisfiable. The solution of the problem at this point is makespan optimal. We improve this approach by setting a lower bound on the makespan derived from the shortest paths from start to goal of each agent. The bound is set to the maximum of all shortest paths. We refer to increments of this lower bound as δ .

Finding a sum-of-costs optimal solution is more involved, as the number of timesteps has to be figured out and the sum-of-costs value has to be restricted. Recall the example in Figure 1 showing that increasing makespan (i.e., the number of timesteps) may decrease the sum-of-costs. There are two main approaches: the *iterative* (Surynek et al., 2016) and the *jump* (Barták and Svancara, 2019) method. The iterative method adds a numerical constraint that bounds the sum-of-costs. Intuitively, this constraint bounds how many extra actions the agents can perform. Specifically, the bound is given by $C_{SOC}(\Pi_{sp}) + \delta$, where Π_{sp} is a possibly conflicting plan consisting of shortest paths for the agents and δ is a number of admissible extra moves. This requires that the number of moves per agent is bounded by the length of its shortest path plus δ . The number of timesteps in which the agent may move is set to $\Pi_{sp} + \delta$. With this setup, the iterative method again follows an iterative deepening approach starting with a δ of zero and increments it until the bounded problem becomes satisfiable. The final solution is sum-of-costs optimal. Algorithm 1 shows the iterative

Algorithm 1: Iterative approach

```

1 iterative (MAPF problem instance)
2    $\delta \leftarrow 0$ ;
3   while No Solution do
4     solve_soc( $\delta$ );
5      $\delta \leftarrow \delta + 1$ ;

```

Algorithm 2: Old jump approach

```

1 jump-old (MAPF problem instance)
2    $\delta \leftarrow 0$ ;
3    $LB(SoC) \leftarrow$  sum of shortest paths;
4   while No Solution do
5      $SoC \leftarrow$  solve_mks( $\delta$ );
6      $\delta \leftarrow \delta + 1$ ;
7    $\delta \leftarrow SoC - LB(SoC)$ ;
8   solve_soc_with_minimization( $\delta$ );

```

approach. Function *solve_soc* takes a MAPF problem bounded by the given δ as described above and solves it.

The *jump* method follows a different approach, as seen in Algorithm 2. First, it starts by finding a makespan optimal solution Π_{mks} . We compute the sum-of-costs $C_{SOC}(\Pi_{mks})$ and use it to find an upper bound on δ . Setting $\delta = C_{SOC}(\Pi_{mks}) - C_{SOC}(\Pi_{sp})$, we do one last call with the properties explained in the iterative approach except that the numerical constraint bounding the total number of moves is replaced by a minimization component. The number of timesteps is again set to $\Pi_{sp} + \delta$. Since δ is an upper bound, the minimization component makes sure that the returned solution is optimal. An important part of this method is that, when finding the makespan optimal solution, we also optimize for the sum-of-costs. This makes the upper bound on the sum-of-costs tighter. Additionally, if $C_{SOC}(\Pi_{mks}) = C_{SOC}(\Pi_{sp})$, we have already found the optimal sum-of-costs solution. Finally, for a makespan optimal solution Π_{mks} with a δ_{mks} and $C_{SOC}(\Pi_{mks}) = C_{SOC}(\Pi_{sp}) + \delta_{mks}$, we have also found the optimal sum-of-costs solution.

A vital enhancement that applies to the above methods is what we call *reachability*, originally introduced in (Surynek et al., 2016) as MDD graphs. Let t_i be the maximum number of moves of an agent a_i with associated start and goal vertices s_i and g_i , and let $dist(u, v)$ be the shortest distance between vertices u and v . We allow the agent to be at a vertex v at timepoint t only if the following conditions hold: $dist(s_i, v) \leq t$ and $dist(v, g_i) + t \leq t_i$. That is, if the vertex v can be reached from the start vertex within t moves and it

is possible to reach the goal vertex from v within the bound on the maximum moves of the agent. As an additional enhancement, we block a vertex v for agents at timepoint t if v corresponds to the goal vertex of another a agent and t is larger than the maximum number of moves of a . This slightly reduces the number of reachable positions if agents have different bounds on the number of maximum moves. Note that this block is implicit in the problem formulation. By adding it to the reachability definition we can avoid grounding and solving effort.

4 ASP ENCODING

We model movements of agents with the encoding shown in Listing 1. Here, we give an intuitive explanation of the encoding. For a more detailed description of ASP’s semantics and syntax, we refer to (Gebser et al., 2015). The encoding assumes as input a MAPF problem given by the facts, over unary and binary predicates *vertex/1* and *edge/2*, respectively, describing the graph’s vertices and edges between them. Additionally, the facts *agent/1*, *start/2*, and *goal/2* provide the agents along with their start and goal vertices. The input must include the facts *delta/1* and *dist/2* or *makespan/1* for sum-of-cost or makespan, respectively. Predicate *dist/2* captures the length of the shortest path between each agent’s start and goal vertices, while *makespan/1* gives the maximum number of moves each agent can perform. Finally, we require facts over *reach/3* that encode the reachability enhancement.

The rules in Lines 1–6 set up the bound on the maximum number of moves of each agent. For the sum-of-cost objective, the horizon of the agents is set individually based on their shortest path and the given δ in Line 2. For the makespan objective, the horizon of all agents is set to the value given by the *makespan* fact on Line 4. Note that we can selectively ground the programs, so Line 2 is never used when solving for makespan, while Line 4 is never used for the sum-of-costs optimization. The choice rule on Lines 8–9 may choose a move for each agent conforming to the reachable positions. Observe that a move to a non-reachable vertex can never happen. The rule in Line 10 sets the positions of agents at the first timepoint to their starting positions. Line 11 sets the position of the agent based on the chosen move while Line 12 keeps the same position if no move was chosen (wait action). Line 14 ensures that the chosen movement starts at the correct vertex. Line 15 makes sure that the agent is never at a position that is not reachable. The rule in Line 16 ensures that

an agent is at exactly one position at every timepoint. Next, the rules in Lines 18 and 19 encode vertex and swapping conflicts, respectively. Finally, Line 20 ensures that an agent is at its goal at the last timepoint.

```

1 #program sum_of_costs.
2 horizon(A,H+D) :- dist(A,H), delta(D).
3 #program makespan.
4 horizon(A,H) :- agent(A), makespan(H).
5 #program mapf.
6 time(A,1..T) :- horizon(A,T).

8 {move(A,U,V,T) : edge(U,V), reach(A,V,T)} 1
9 :- reach(A,U,T-1).
10 at(A,V,0) :- start(A,V), agent(A).
11 at(A,V,T) :- move(A,_,V,T).
12 at(A,V,T) :- at(A,V,T-1), not move(A,V,_,T),
13         time(A,T).
14 :- move(A,U,_,T), not at(A,U,T-1).
15 :- at(A,V,T), not reach(A,V,T).
16 :- {at(A,V,T)} != 1, time(A,T).

18 :- {at(A,V,T)} > 1, vertex(V), time(_,T).
19 :- move(_,U,V,T), move(_,V,U,T), U<V.
20 :- goal(A,V), not at(A,V,H), horizon(A,H).

```

Listing 1: ASP encoding for bounded MAPF.

The encoding is sufficient to find makespan optimal solutions. However, for the sum-of-cost objective, we must add additional rules. We assign penalties to agents not at their goal position with the following rules:

```

penalty(A,N) :- dist(A,N+1), N>=0.
penalty(A,T) :- dist(A,N), at(A,U,T),
                not goal(A,U), T>=N.
penalty(A,T) :- penalty(A,T+1), T>=0.

```

The first rule applies a penalty to every timepoint below the shortest path of the agent. At higher timepoints, penalties are applied if agents are not at their goal positions. Finally, if a penalty was applied at any timepoint, then every previous timepoint must also have a penalty. The sum of penalties corresponds to the cost of a solution.

For the iterative approach, we must also add the numerical constraint described in Section 3. The following two rules calculate the bound based on the shortest path of each agent and the given δ and enforce that the sum of costs is below the bound:

```

bound(H+D) :- H=#sum{T,A: dist(A,T)}, delta(D).
:- #sum{1,A,T: penalty(A,T)} > B, bound(B).

```

For the jump approach, we simply minimize the number of penalties using Clingo’s built-in optimization support:

```

#minimize{1,A,T : penalty(A,T)}.

```

Algorithm 3: New jump approach

```

1 jump (MAPF problem instance)
2    $\delta \leftarrow 0$ 
3    $LB(SoC) \leftarrow$  sum of shortest paths
4   while No Solution do
5      $SoC \leftarrow$ 
6     | solve_soc_no_numerical_constraint( $\delta$ )
7     |  $\delta \leftarrow inc\_delta(\delta)$ 
8    $\delta \leftarrow SoC - LB(SoC)$ 
9   solve_soc_with_minimization( $\delta$ );

```

5 IMPROVING THE JUMP APPROACH

In this section, we consider a refinement of the jump model presented in Section 3. We begin by noting that the first step of finding a makespan optimal solution is unnecessarily expensive. Since the main reason to find this initial solution is to get an upper bound on the sum of costs, any solution works. This leads us to the first and most important enhancement. Instead of finding a makespan optimal solution, we use the *iterative* approach, disregarding the numerical constraint, to find an initial solution. The reasoning behind this is that the subproblems that the iterative approach needs to solve are easier than finding a makespan optimal solution. This is because the maximum number of moves of the agents is bound by the length of their shortest path and not by the length of the longest shortest path of all agents. Consequently, the resulting compilation is smaller and the solving time is reduced. This is combined with the reachability enhancement, where the reduced movement range of the individual agents leads to fewer reachable vertices. The potential drawback is that, due to the reduced movement range of the agents, it may need a higher δ to find a solution. However, since the problems are easier to solve, we expect that even with the higher number of solver calls, the initial solution is found much faster.

In the original jump and iterative approaches, δ is always increased by one to guarantee that the returned solution is optimal. However, we can overshoot the bound as we do not need an optimal solution here. This allows us to increase δ in different ways. We introduce two ways to increase δ : a multiplicative increase where δ is multiplied by a constant factor, and an additive increase where δ is increased by a constant value.

The improved jump approach is given in Algorithm 3. Notice that the only differences to the original jump algorithm are Lines 5 and 6.

6 BENCHMARKS

We run our experiments on the set of instances stemming from (Husár et al., 2022). The benchmark has maps with layouts *random*, *room*, *maze*, and *empty* and sizes 16×16 , 32×32 , 64×64 and 128×128 . All maps start with 5 agents and the number of agents increases by 5 up to 100 agents. The instances are also divided into two categories depending on the length of the shortest path of the agents. For instances of type *condensed* the shortest path length of all agents is almost the same. Specifically, for a given instance we select a length L . The shortest path of the agents is then within five percent of L . For instances of type *uneven*, the shortest path length of the agents is randomized.

We consider the following functions in Line 6 of Algorithm 3 to increase the δ aside from the basic increase by one:

- $\delta \leftarrow \delta + 2$
- $\delta \leftarrow \delta + 5$
- $\delta \leftarrow \delta * 1.5$
- $\delta \leftarrow \delta * 2$

Finally, we compare the default optimization strategy (branch-and-bound) with a strategy based on unsatisfiable cores (Andres et al., 2012).

6.1 Results

All benchmarks were run using Clingo 5.6.2 on an Intel Xeon E5-2650v4 under Debian GNU/Linux 10, with a timeout of 300 seconds and a memory limit of 28 GB.

In Table 1, we see the results for the different δ increase strategies. The columns report the number of instances solved for a given size with the total number of instances in parenthesis. Although all strategies perform similarly, we see that the increase by two is the best strategy for the jump approach. Notably, the increase by five is the worst strategy. This suggests that an overly large increase in δ is detrimental to the performance of the jump approach. The bigger the jump, the bigger the possible overshoot of the minimum δ needed to find the first solution. This might lead to a significant enough overhead to negate the advantage of skipping some solver calls. Hence, in the following discussion, we only include results for the default increase and the increase by two.

Table 2 shows the total number of instances solved, separated by size and type. First, we mention that the chosen optimization strategy has a significant impact on the performance of the jump approach, old and new. The use of unsatisfiable cores is significantly faster than the branch-and-bound approach. The approaches

using unsatisfiable cores managed to solve around 100 more instances than when using branch-and-bound. For this reason, we only consider the results using unsatisfiable cores in the following discussion.

Next, we observe that the *jump-old* approach is better than the *iterative* approach for instances of size 16×16 and 32×32 of type uneven, which is consistent with the results of (Barták and Svancara, 2019). However, for the instances of size 64×64 and 128×128 , the performance of the *jump-old* approach quickly deteriorates. This is where the new approach showcases its strength. We see that it has a similar performance to the *jump-old* approach on the smaller instances while being significantly better on the larger instances. Additionally, it outperforms the *iterative* approach on all instance sizes. We also note that the enhancement of the *jump+2* approach, where δ increases by two, provides a small boost in performance. The other strategies to increase δ similarly provide a small boost in performance, although, an increase of two seems to be the best.

Finally, we comment on the instances of type condensed. Since all agents have a similar shortest path length, the reachability calculation is almost the same regardless of the objective function. Hence, for these kinds of instances, there is no difference between the old and new approaches. The results confirm this observation. The very slight improvement from the new *jump* approach is because the condensed instances have very slight differences in the shortest path length of the agents.

Table 3 shows the average number of reachable positions and the average number of calls made to the solver. We observe how the *iterative* approach always has the highest number of calls, followed by *jump* and *jump-old*, in that order. From these numbers, one could conjecture that the *jump-old* approach is the best since it has to solve fewer subproblems. However, looking at the number of reachable positions, we can see that although it has to solve fewer subproblems, they are much more difficult. The trend of the reachable positions closely resembles the trend of the total number of instances solved. This is because the number of reachable positions is a good indicator of the difficulty of the problem. Since the final problems that the *jump* and *jump-old* approaches have to solve are likely the same, the fact that the new approach has an easier time finding the first solution is the key to its success. We also note that the results are similar no matter the map layout.

Table 1: Results for all instances grouped by size for the different δ increase strategies. The columns report the number of instances solved with the total number of instances in parenthesis.

| | | jump | jump+2 | jump+5 | jump*1.5 | jump*2 |
|------------------|---------------|-------------|---------------|---------------|-----------------|---------------|
| 16×16 | (400) | 275 | 275 | 274 | 274 | 273 |
| 32×32 | (400) | 371 | 370 | 371 | 372 | 371 |
| 64×64 | (400) | 464 | 464 | 462 | 465 | 462 |
| 128×128 | (400) | 341 | 346 | 338 | 342 | 342 |
| Total | (3200) | 1451 | 1455 | 1445 | 1453 | 1448 |

Table 2: Results for all instances grouped by size. The columns report the number of instances solved with the total number of instances in parenthesis.

| | | iterative | unsatisfiable core | | | branch-and-bound | | |
|------------------------|---------------|------------------|---------------------------|-------------|---------------|-------------------------|-------------|---------------|
| | | | jump-old | jump | jump+2 | jump-old | jump | jump+2 |
| | | condensed | | | | | | |
| 16×16 | (400) | 124 | 137 | 137 | 137 | 127 | 127 | 127 |
| 32×32 | (400) | 145 | 161 | 163 | 162 | 144 | 145 | 145 |
| 64×64 | (400) | 158 | 184 | 184 | 184 | 159 | 159 | 159 |
| 128×128 | (400) | 117 | 137 | 139 | 139 | 122 | 128 | 128 |
| Total condensed | (1600) | 544 | 619 | 623 | 622 | 552 | 559 | 559 |
| | | uneven | | | | | | |
| 16×16 | (400) | 129 | 137 | 138 | 138 | 127 | 127 | 128 |
| 32×32 | (400) | 174 | 194 | 208 | 208 | 156 | 190 | 190 |
| 64×64 | (400) | 230 | 107 | 280 | 280 | 92 | 247 | 247 |
| 128×128 | (400) | 184 | 6 | 202 | 207 | 6 | 194 | 200 |
| Total uneven | (1600) | 717 | 444 | 828 | 833 | 381 | 758 | 765 |
| Total | (3200) | 1261 | 1063 | 1451 | 1455 | 933 | 1317 | 1324 |

Table 3: Results for all instances solved by all approaches using unsatisfiable core optimization grouped by size. The columns report the average cummulative reachable positions in thousands, and the average calls made to the solver in parenthesis.

| | iterative | jump-old | jump | jump+2 |
|------------------------|-------------------|------------------|-----------------|-----------------|
| condensed | | | | |
| 16×16 | 67 (12.6) | 19 (6.2) | 19 (6.2) | 19 (5.3) |
| 32×32 | 82 (13.4) | 23 (6.3) | 23 (6.7) | 21 (5.5) |
| 64×64 | 122 (11.7) | 42 (5.5) | 37 (6.5) | 37 (5.5) |
| 128×128 | 184 (10.5) | 92 (4.0) | 76 (6.0) | 75 (5.1) |
| Total condensed | 112 (12.1) | 42 (5.6) | 37 (6.4) | 37 (5.4) |
| uneven | | | | |
| 16×16 | 120 (17.4) | 44 (3.7) | 25 (9.2) | 23 (6.7) |
| 32×32 | 175 (11.6) | 425 (3.6) | 57 (6.6) | 52 (5.3) |
| 64×64 | 100 (8.4) | 910 (3.4) | 40 (5.9) | 39 (4.9) |
| 128×128 | 1 (3.0) | 270 (3.0) | 1 (3.0) | 1 (3.0) |
| Total uneven | 137 (12.5) | 430 (3.5) | 42 (7.2) | 39 (5.6) |
| Total | 123 (12.3) | 211 (4.7) | 39 (6.7) | 38 (5.5) |

7 CONCLUSION

While finding a makespan optimal solution is quite straightforward, there have been attempts to refine the algorithm to improve performance (Husár et al., 2022). The sum-of-cost objective requires more complicated algorithms as we have bounds on the total cost of the plan, as well as on the maximum makespan of the agents. Algorithms to find the sum-of-cost optimal solutions for reduction-based solver were first conceived for SAT in (Surynek et al., 2016; Barták and Svancara, 2019). Later, the jump approach was implemented for ASP in (Gómez et al., 2021), however, the paper focused mostly on improving the encoding.

In this paper, we have presented a new approach to find a sum-of-cost optimal solution in reduction-based solvers. The new approach combines the advantages of both previously known algorithms. It makes use of the reduced search space of the *iterative* approach, while “jumping” to a δ that guarantees the existence of a solution, similarly to the old *jump* method. Our experiments show that the new approach is better on all instance sizes. Additionally, we provide data that highlights the importance of the optimization strategy used in the solver. Lastly, we remark that, in practice, agents usually do not have similar shortest path lengths. This means the instances of type uneven, where the best results are seen, are the most realistic.

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